

CERTAIN RESULTS ON NON-ASSOCIATIVE RINGS

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ABSTRACT: In this paper mainly we have obtained certain theorems on non-associative rings to be commutative under multiplication.

KEYWORDS: non-associative ring, 2-divisible associative ring

INTRODUCTION: M.Ashraf, M.A. Quadri and D-Zelinsky [] generalized some results for non associative rings. In that paper they obtained that for a non associative ring in which $(xy)^2 - yx^2y$ is centre then R is commutative. In this paper we proved that for a non associative ring R with unity 1 such that $(xy)^2 = (yx^2)y$ then R is commutative. It is also observed that for a non associative ring R with unity 1 satisfying $x^2y = yx^2$ then R is commutative. It is also observed in this paper that for 2-Divisible associative ring R with unity 1 such that $(x, x^2 + y^2) = 0$ then R is commutative.

Definition 1: A non empty set R together with the binary operations +, . satisfying

- 1) $(R, +)$ is a commutative group
- 2) R satisfies Distributive laws then $(R, +, \cdot)$ is called a non associative ring.

Definition 2: A non associative ring R is said to be 2-Divisible ring if $x(x^2y^2 + xy) = (x^2y^2 + xy)x$.

From the following theorem it is observed that for a non associative ring with unity satisfying the condition $(xy)^2 = (yx^2)$ is commutative

Theorem 1: If R is a non associative ring R with unity 1 satisfy the condition which $(xy)^2 - yx^2y$ then R is commutative.

Proof: Let R be a non associative ring R with unity 1 satisfy the condition which $(xy)^2 - yx^2y$ then by replacing x by x+1 then we have

$$\begin{aligned} [(x+1)y]^2 &= [y(x+1)]^2 y \Rightarrow [(xy+y)]^2 = [y(x+1)(x+1)]y \\ &\Rightarrow [(xy+y)](xy+y) = [y(xx+x+x+1)]y \\ &= (yxx+yx+yx+y)y \\ &= yxxy+yxy+yxy+yy \\ &\Rightarrow xyxy+xyy+yxy+yy = yx^2y+yxy+yxy+yy \end{aligned}$$

Similarly by replacing y by y+1 then we have

$$\begin{aligned}
[x(y+1)]^2 &= [(y+1)x^2]y+1 \\
\Rightarrow [xy+x]^2 &= (yx^2+x^2)(y+1) \\
\Rightarrow (xy+x)(xy+x) &= yx^2y+yx^2+x^2y+x^2 \\
\Rightarrow xyxy+xyx+xyx^2 &= yx^2y+yx^2+x^2y+x^2
\end{aligned}$$

Imply that R is commutative.

Theorem2: If R is a non associative ring R with unity 1 satisfy the condition which $x^2y=yx^2$ then R is commutative.

Proof: Let R be a non associative ring R with unity 1 satisfy the condition $x^2y=yx^2$

Replacing x by x+1 then $(x+1)^2y=y(x+1)^2$

$$\Rightarrow [(x+1)(x+1)]y=y[(x+1)(x+1)]$$

$$\Rightarrow (x^2+x+x+1)y=y(x^2+x+x+1)$$

$$\Rightarrow x^2y+xy+xy+y=yx^2+yx+yx+y$$

Similarly by replacing y by y+1 then

$$x^2(y+1)=(y+1)x^2$$

$$\Rightarrow x^2y+x^2=yx^2+x^2$$

Imply that R is commutative.

Theorem3: If R is a 2-divisible non associative ring R with unity 1 such that $(x, x^2y^2+xy)=0$ then R is commutative.

Proof: If R is a 2-divisible non associative ring R with unity 1 satisfy the condition $(x, x^2y^2+xy)=0$

Imply that $x(x^2y^2+xy)=(x^2y^2+xy)x$

Replacing x by x+1 then

$$(x+1)[(x+1)^2y^2+(x+1)y]=[(x+1)^2y^2+(x+1)y]x$$

Simplifying

$$\text{Similarly Replacing } y \text{ by } y+1 \text{ then } x[x^2(y+1)^2+x(y+1)]=[x^2(y+1)^2+x(y+1)]x$$

Simplifying then it is easily observed that R is commutative.

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